

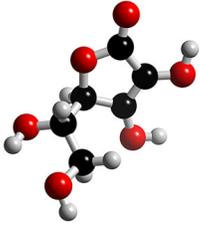
# Counting

Permutations | Combinations

It is expensive and far from logical to proceed through scientific discovery by chance.

Imagine for human health purposes, you need to test and experiment with all possible bi-products of this molecule of Vitamin C (C<sub>6</sub>H<sub>8</sub>O<sub>6</sub>) in case it reacts to the body and breaks down. How many elements and molecules do you need to test?

How is this different from what you have already learned?



# Counting

Permutations | Combinations

Permutations	Combinations
<p><b>Permutation:</b> the <b>order</b> of the events is important and it <b>matters</b> which item is placed first. <b>Ranking, matching</b> or <b>sequencing</b> elements or groups to specific identities or positions is also a permutation.</p> <ul style="list-style-type: none"> <li>-arranging all elements</li> <li>-arranging some elements</li> <li>-sum or product rule</li> <li>-repetition of elements</li> <li>-identical elements</li> <li>-stationary elements</li> <li>-direct &amp; indirect method</li> </ul>	<p><b>Combination:</b> the <b>order</b> of the events doesn't matter and it <b>does not matter</b> which item is placed first.</p> 

Permutations

Results: ABC, ACB, BAC, CAB, CBA, CAB

Combinations

$$\frac{n!}{(n-r)!} = {}_n P_r \quad \quad {}_n P_r \div r! = {}_n C_r$$

How does this image explain to us the connection between permutations and combinations?

### Combinations

derivation & formula | no restrictions | patterns of choice | restrictions | any or all options | combinatorics

What is the connection between permutations and combinations?

How can we modify the permutation formula for combinations?

$${}^n C_r = \frac{{}^n P_r}{r!} = \frac{n!}{(n-r)! r!}$$

Explanation: In calculating the permutations of  $n$  select  $r$  objects, we also considered the number of ways in which we could arrange the  $r$  objects. Since  $r$  objects can be arranged in  $r!$  ways, we divide by this number.

### Practice

- If this class has 25 students, how many different ways can you break into groups of 4?
- The Math Club has 15 members.
  - In how many ways can an executive President, Vice-President, and Secretary be chosen?
  - How many executive councils are there?
- The Junior Boys Volleyball team has six members.
  - How many different starting line-ups can be selected?
  - What if the position of the player mattered?
  - How many line-ups if the team consisted of 10 people?
- How many different groups of three toys can a child choose to take on vacation if the toy box contains 10 toys?
- In how many ways can a teacher select five students from the class of 30 to have a detention?

### Combinations

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Do combinations have a pattern? Why?

Students to be selected	Number of Groups
2 students	
3 students	
4 students	
5 students	
6 students	
7 students	
8 students	

Eight student express an interest in taking part in an exchange to Germany.

How many different groups could go if as Principal you selected:

Use this pattern to predict the number of ways one chairperson can be chosen from 8 students. How about zero chairs?

### Combinations: Pascal's Triangle

derivation & formula | no restrictions | patterns of choice | restrictions | any or all options | combinatorics

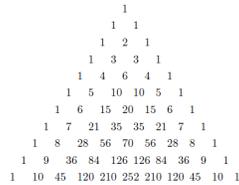
How many patterns can you find?  
How is this connected to combinations?

				1						
				1	1					
			1	2	1					
		1	3	3	1					
	1	4	6	4	1					
1	5	10	10	5	1					
1	6	15	20	15	6	1				
1	7	21	35	35	21	7	1			
1	8	28	56	70	56	28	8	1		
1	9	36	84	126	126	84	36	9	1	
1	10	45	120	210	252	210	120	45	10	1

### Practice

Do these questions again, but use pascal's triangle to answer them.

3. The Junior Boys Volleyball team has six members. In how many ways can a starting line-up be chosen?
4. How many different groups of three toys can a child choose to take on vacation if the toy box contains 10 toys?
5. In how many ways can a teacher select five students from the class of 30 to have a detention?

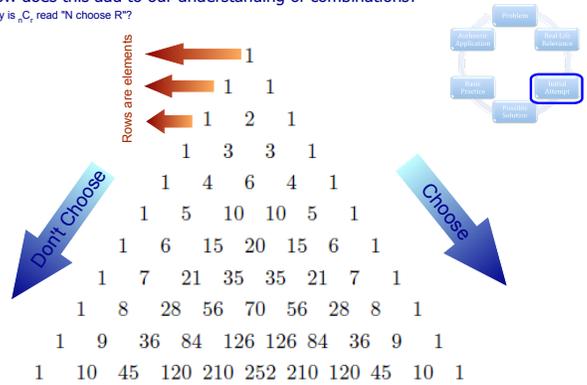


### Combinations: Pascal's Triangle

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How does this add to our understanding of combinations?

Why is  ${}_nC_r$  read "N choose R"?



### Combinations: Pascal's Triangle

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What does the sum of a row in pascal's triangle represent? Why?

How many groups can be formed from three people?



### Practice

6. Determine the row number for each of the following row sums from Pascal's triangle.

- a) 256
- b) 2048
- c) 16384
- d) 65536

7. Determine the sum of the terms in each of these rows in Pascal's triangle.

- a) Row 12
- b) Row 20
- c) Row 25
- d) Row  $(n - 1)$

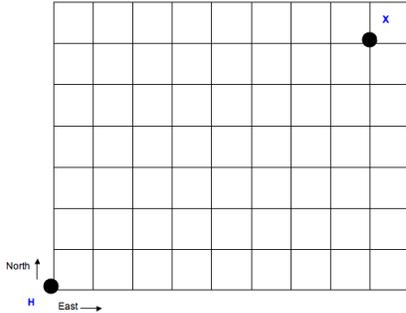
8. In how many ways can Kimberly choose to invite her seven friends over for a sleepover assuming that she has to invite at least one friend over?

9. Why is  ${}_6C_7$  not possible?



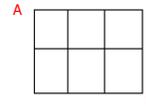
The Case of the Stolen Jewels

10. Here is a street map of part of the city of London. Inspector Canard's next case involved a million dollars worth of jewellery stolen from a hotel suite in the city. This map shows the hotel marked with the letter H. Inspector Canard is certain that the thieves and the jewels are located at the spot marked by the letter X. In order to catch the thieves, Canard must determine all the possible routes from H to X. The inspector is driving and all the streets are one-way going north or east. How many different routes do you think Inspector Canard has to check out?



Pathfinders

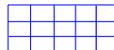
11.



- Count and draw the number of paths from A to B by only going south or east.
- Starting at corner A begin placing Pascal's Triangle. At each successive corner continue with Pascal's Triangle pattern until corner B. How does the number at corner B relate to the number of paths you found in part a?
- If  $n$  represents the number of rows plus the number of columns (in grid AB) and  $r$  represents the number of rows or columns, find  $C(n, r)$ .

12. Solve the following problems using both Pascal's Triangle and/or Combinations.

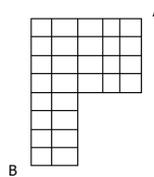
- A school is 5 blocks west and 3 blocks south of a student's home. How many different routes could the student take from home to school by going west or south at each corner. Draw a diagram.



- In the following arrangements of letters start at the top and then proceed to the next row by moving diagonally left or right. Determine the number of different paths that would spell the word PERMUTATION.

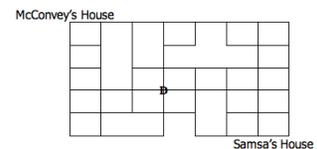


c.



Find the number of paths from point A to Point B by only going south or west.

- Mr. Samsa wishes to visit Mr. McConvey by traveling only north or west. If he has to pick up Mr. Denhart (who lives at the intersection marked "D"), how many different paths can Samsa travel?



13. Fill in the missing numbers using Pascal's method.

_____	_____	495	_____
_____	3003	_____	825
_____	_____	2112	_____

14. In the following arrangements of letters, start from the top and proceed diagonally left or right. How many different paths will spell each word?

a)

b)

c)

15. The first nine terms of a row of Pascal's triangle are shown below. Determine the first nine terms of the previous and next rows.

1 16 120 560 1820 4368 8008 11440 12870

16. Determine the number of possible routes from A to B if you travel only south or east.

a)

b)

c)

17. Sung is three blocks east and five blocks south of her friend's home. How many different routes are possible if she walks only west or north?

18. Ryan lives four blocks north and five blocks west of his school. Is it possible for him to take a different route to school each day, walking only south and east? Assume that there are 194 days in a school year.

### Combinations

derivation & formula | no restrictions | patterns of choice | restrictions | any or all options | combinatorics

**What if you are not able to use all the elements to form groups?**

Eight students express an interest in taking part in an exchange to Germany.

**Restriction:** There's only accommodation for six (two rooms of three each). If six of eight interested students are female, and you can't mix genders in a room, how many different groups of students can go?

### Practice

19. A committee of three is to be formed from five Math teachers and four English teachers. In how many ways can the committee be formed if there:

- a. are no restrictions
- b. must be one math teacher
- c. must be one English teacher
- d. must be only math teachers

20. How many ways committee of 7 can be chosen from 16 males and 10 females if

- a. there are no restrictions?
- b. they must be all females?
- c. they must be all males?

21. In how many ways can six players be chosen from fifteen players for the starting line- up

- a. if there are no restrictions
- b. if Jordan must be on the starting line.
- c. if Tanvir has been benched and can't play.

22. How many full houses are there in poker that contain at least two black cards?

### Combinations

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James found 10 shirts, 4 pairs of pants, and 3 hats from a dumpster. How many different outfits can he make?

How is this like exercise 16 from the permutations note?



23. Imagine for human health purposes, you need to test and experiment with all possible bi-products of this molecule of Vitamin C ( $C_6H_8O_6$ ) in case it reacts to the body and breaks down. How many elements and molecules do you need to test? ?

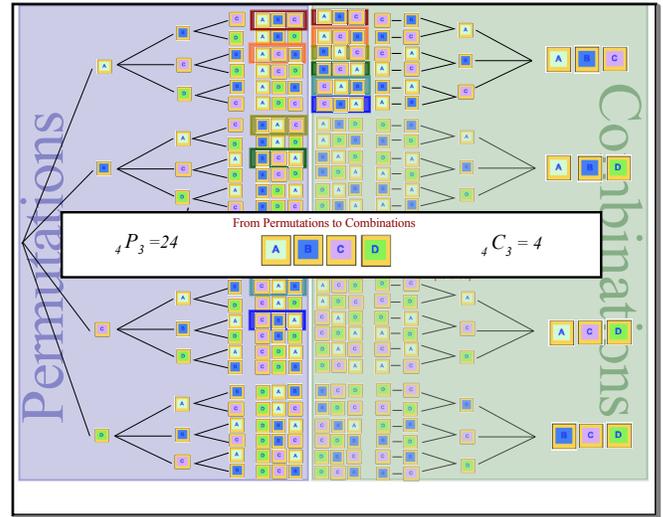
24. Medical researchers are testing the effects of particular options for effective treatment of advanced skin cancer. In the first stage one can receive either radiation treatment, internal chemo therapy, topical chemo, or surgery to remove the malignant tissue. The second stage can be either more radiation, either form of chemo, or naturopathic methods. Each stage can involve hospitalization or immediate return to home after a day procedure. How many possible treatments are available for study?

## Combinations

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A bridge hand consists of 13 cards. How many bridge hands include 4 cards of one suit, 7 of another, and 2 of a third?

How is this different from what you've learned so far?



### Answer Clues

- |                                  |                     |
|----------------------------------|---------------------|
| 1. 12,650                        | 12a). 56            |
| 2a). 2,730                       | b). 252             |
| b). 455                          | c). 756             |
| 3a). 1                           | d). 56              |
| b). 720                          | 13. Top left #: 924 |
| c). 210                          | 14a) 128            |
| 4. 120                           | b) 972              |
| 5. 142,506                       | c) 80               |
| 6a). 8                           | 15.                 |
| b). 11                           | 16a). 126           |
| c). 14                           | b). 1,301           |
| d). 16                           | c). 136             |
| 7a). 4,096                       | 17. 56              |
| b). 1,048,576                    | 18. 126 - no        |
| c). 33,554,432                   | 19a). 84            |
| d). $2^{(n-1)}$                  | b). 30              |
| 8. 127                           | c). 40              |
| 9. Group of 7 with only 6 things | d). 10              |
| 10. 3,003                        | 20a). 657,800       |
| 11a). 10 ways                    | b). 120             |
|                                  | c). 11,440          |
|                                  | 21a). 5,005         |
|                                  | b). 2,002           |
|                                  | c). 3,003           |
|                                  | 22. 3432            |
|                                  | 23. 439             |
|                                  | 24. 64              |

## Quiz

Answer the following questions and submit. You have until 10:10am.

22. A committee of three is to be formed from five Math teachers and four English teachers. In how many ways can the committee be formed if there:
- are no restrictions
  - must be one math teacher
  - must be one English teacher
  - must be only math teachers
  - two must be from one department, and one from another
25. How many full houses are there in poker that contain at least two black cards?