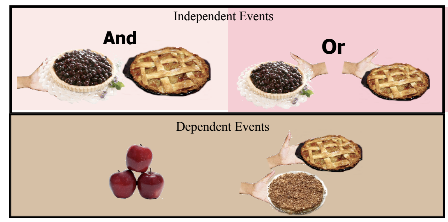
You have a die and a deck of cards.
What is the probability of rolling a six and then drawing a six?

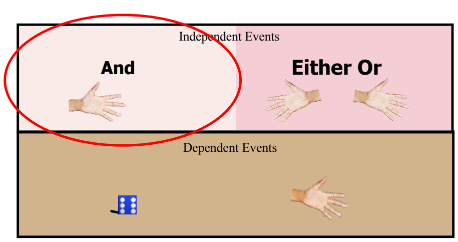
Introductory Probability

Types & terminology | Basic formula | Activating prior learning | Sum and product rule | Independent AND| OR | Venn Diagrams | Dependent Probability



Introductory Probability

Types & terminology | Basic formula | Activating prior learning | Sum and product rule | Independent AND| OR | Venn Diagrams | Dependent Probability



"And" = Multiply

Given: two independent events A and B. Each event has a certain probability of occurring.

Wanted: the probability of both events occurring (i.e. A "And" B occurring).

$$P(A \cap B) = P(A) \times P(B)$$

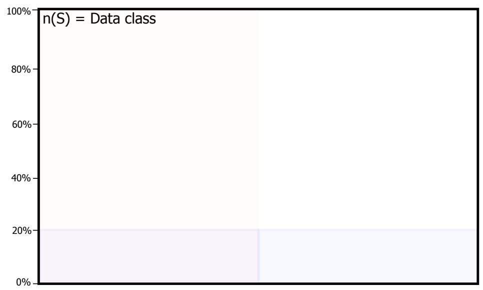
Condition: A and B are independent - that is, the outcome of one event does not impact the outcome of the other.

"And" = Multiply

So, why does this formula work?

P(Female ∩ Blue Eyes) = P(Female) x P(Blue Eyes)

n(F) = 50%   n(B) = 20%



"And"

Example 1: A game consists of rolling a die, then cutting a deck of cards. Find the probability that a six is rolled and a six is cut from the deck of cards.

The event A is to roll a six with a die.
The event B is to cut a six from the deck.
Note that the two events are independent; the occurrence of one does not effect the occurrence of the other.

First calculate the probability of **rolling a six with a die.**

$n(A) = 1$

$n(S) = 6$

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{6}$

Now calculate the probability of **cutting a six from a deck of cards.**

$n(B) = 4$

$n(S) = 52$

$P(B) = \frac{n(B)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

To find the probability of both events occurring, multiply their individual probabilities.

$$P(A \cap B) = P(A) \times P(B)$$
$$= \frac{1}{6} \times \frac{1}{13}$$
$$= \frac{1}{78}$$

The probability of rolling a six with a die and cutting a six from the deck of cards is $\frac{1}{78}$.

"And"

Example 2: Michael estimates that his probability of passing Mathematics is 0.8 and his probability of passing English is 0.9. Find the probability that Michael

a. will pass both Mathematics and English.
b. will pass Mathematics but will fail English.
c. will fail Mathematics but pass English.
d. will not pass either Mathematics or English.

Let A be the event that Michael passes Mathematics and A′, the complement, that he fails.
Let B be the event that Michael passes English and B′, the complement, that he fails.
Assume that A and B are independent!

Given: P(A) = 0.8
P(B) = 0.9

a. Find P(A and B)

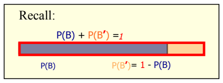
$$P(A \cap B) = P(A) \times P(B)$$
$$= 0.8 \times 0.9$$
$$= 0.72$$

The probability that Michael passes both Mathematics and English is equal to 0.72.

b. Find P(A and B′)

$$P(A \cap B') = P(A) \times P(B')$$
$$= P(A) \times [1 - P(B)]$$
$$= 0.8 \times [1 - 0.9]$$
$$= 0.8 \times 0.1$$
$$= 0.08$$

Recall:



The probability that Michael will pass Mathematics and fail English is equal to 0.08.

"And"

c. Find P(A′ and B)

$$P(A' \cap B) = P(A') \times P(B)$$
$$= [1 - P(A)] \times P(B)$$
$$= [1 - 0.8] \times 0.9$$
$$= 0.2 \times 0.9$$
$$= 0.18$$

The probability that Michael fails math and passes English is equal to 0.18.

d. Find P(A′ and B′)

$$P(A' \cap B') = P(A') \times P(B')$$
$$= [1 - P(A)] \times [1 - P(B)]$$
$$= [1 - 0.8] \times [1 - 0.9]$$
$$= 0.2 \times 0.1$$
$$= 0.02$$

The probability that Michael fails both courses is equal to 0.02.

"And"

Summarized in a table...

Outcome	Probability
Pass both Math and English $P(A \cap B)$	0.72
Pass Math, Fail English $P(A \cap B')$	0.08
Fail Math, Pass English $P(A' \cap B)$	0.18
Fail both courses $P(A' \cap B')$	0.02

0.72 + 0.08 + 0.18 + 0.02 = 1.00

Does this make sense?

Introductory Probability

Types & terminology | Basic formula | Activating prior learning | Sum and product rule | Independent AND| OR | Venn Diagrams | Dependent Probability

How can we combine independent probability calculations with tree diagrams?

Try it for the last question.

1. Four friends, two females and two males, are playing contract bridge. Partners are randomly assigned for each game. What is the probability that the two females will be partners for the first game?

2. Amitesh estimates that he has a 70% chance of making the basketball team and a 20% chance of having failed his last geometry quiz. He defines a "really bad day" as one in which he gets cut from the team and fails his quiz. Assuming that Amitesh will receive both pieces of news tomorrow, how likely is it that he will have a really bad day?

3. In the popular dice game Yahtzee ®, a Yahtzee occurs when five identical numbers turn up on a set of five standard dice. What is the probability of rolling a Yahtzee on one roll of the five dice?

4. A fruit basket contains five red apples and three green apples. You randomly select one apple, return it, and select a second apple. What is the probability that
   a) you will select two red apples?
   b) You will not select two green apples?

5. There are two tests for a particular antibody. Test A gives a correct result 95% of the time. Test B is accurate 89% of the time. If a patient is given both tests, find the probability that
   a) both tests give the correct result
   b) neither test gives the correct result
   c) at least one of the tests gives the correct result

Answer Key

1. 1/3
2. 6%
3. 6/7776
4a). ~39%
  b). ~85.9%
5a). 84.55%
  b). 0.55%
  c). 99.45%