Probability Distributions:

Geometric Distribution

Imagine a binomial distribution, but you're only allowed one success (P), and then you have to stop.

If you fail your G2 twice, and then pass...

If you hit your snooze button 8 times, and then finally wake up...

If you call your ex-girlfriend 16 times, and she finally answers her cell...

If you ask Mr. Gallant or Mr. Boulton 32 times for extra marks, and we finally agree....

Then you have participated in a geometric distribution.

$$Q \times Q \times Q \ldots Q \times Q \times P$$

or

$$Q^{n-1}P$$

So when are you allowed to be mad?

$$E(X) = \frac{Q}{P}$$

What is the expected number of failures when trying to roll a 3 with a six-sided die?

Probability of Success $\frac{1}{6}$ — six rolls, 1 success 5 failures — Probability of Failure $\frac{5}{6}$

...or 5 failures per success, right?

$$E(X) = \frac{5/6}{1/6} = \frac{5}{1} = 5$$

1. Which of the following situations is modeled by a geometric distribution? Explain your reasoning.
   a) rolling a die until a 6 shows
   b) counting the number of hearts when 13 cards are dealt from a deck
   c) predicting the waiting time when standing in line at a bank
   d) calculating the probability of a prize being won within the first 3 tries
   e) predicting the number of successful launches of satellites this year

2. For a 12 sided die,
   a) what is the probability that the first 10 will be on the third roll?
   b) what is the expected waiting time until a 1 is rolled?

3. To finish a board game, Sarah needed to land on the last square by rolling a sum of 2 with two dice. She was dismayed that it took her eight tries. Should she have been surprised? Explain.

4. A teacher provides a pizza for his class if they earn an A-average on any test. The probability of the class getting an A-average on one of his tests is 8%.
   a) What is the probability that the class will earn a pizza on the fifth test?
   b) What is the probability that the class will not earn a pizza for the first seven tests?
   c) What is the expected waiting time before the class gets a pizza?

5. Minh has a summer job selling replacement windows by telephone. Of the people he calls, nine out of ten hang up before he can give a sales pitch.
   a) What is the probability that, on a given day, Minh's first sales pitch is on his 12^{th} call?
   b) What is the expected number of hangups before Minh can do a sales pitch?

6. Despite its name, Zippy Pizza delivers only 40% of its pizzas on time.
   a) What is the probability that its first four deliveries will be late on any given day?
   b) What is the expected number of pizza deliveries before one is on time?

7. In repeated rolling of a pair of dice,
   a) What is the probability that the first roll of doubles occurs on the third roll?
   b) What is the expected waiting time (number of rolls) before you roll doubles?

8. In a gambling game, Simon tosses a coin until a head is uppermost. He then receives $2n, where n is the number of tosses.
   a) What is the probability that Simon receives $8.00 in one play of the game?
   b) If the player must pay $5.00 to play, what is the win/loss per game?

Answer Clues

1a) yes
b) no - binomial probability
c) yes
d) no - binomial probability
e) no - binomial expected value

2a) 7%
b) 11 rolls

3) no; 35 is most likely.

4a) 5.7%
b) 4.5%
c) over 11 tests

5a) 3.1%
b) 9 hang ups

6a) 5.2%
b) one and a half

7a) 11.6%
b) 5

8a) 6.3%
b) player expects to lose $1 per game