

Authentic Probability Tasks I: Make a Game!

Binomial | Geometric | Hypergeometric

- 1 Find a partner
- 2 Design a game
Plan well, because:
Must be in excel
Must contain a probability distribution
- 3 Design a probability question
It will likely be Expected value/cost of playing
(Hence the importance of design)
- 4 Play someone's game
Play someone else's
Ask some questions
Then pick ONE game
(no one else can pick it)
- 5 Answer the probability question
Give your solution to the game designers to evaluate

Game/Answers evaluated for:
Problem solving
Creative design
Effective distribution incorporation
Identifying probability types
Approach (logical, clever)
Not necessarily correctness

Authentic Probability Tasks II: Real life problems

Binomial | Geometric | Hypergeometric

How is this stuff used in the real world?



Engineering/Manufacturing

1. Suppose that 5% of the first batch of engines off a new production line have flaws. An inspector randomly selects six engines for testing.
 - a) What's the **probability that two of the engines are flawed?**
 - b) What is the **expected number of flawed engines in the sample.**
 - c) In a sample of 600 engines, 60 are flawed in some way. What's the probability of this? What conclusion can the inspector make?
2. A certain type of rocket has a failure rate of 1.5%.
 - a) Design a simulation to illustrate the expected number of failures in 100 launches.
 - b) Use the methods developed in this section to determine the probability of more than 4 failures in 100 launches.
 - c) What is the **expected number of failures in 100 launches of the rocket?**


• Make Decisions
• Analyze and assess
• Determine likely outcomes
• Expected costs and values
Apply to Life

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Marketing Research


3. Suppose that 65% of the families in a town own computers. If eight families are surveyed at random,
 - a) what is the **probability that four own computers?**
 - b) what is the **expected number of families with computers?**
 - c) if, for purposes of eliminating statistical discrepancy, you wanted to ensure you surveyed at least 100 random families who own computers, how many cold-calls would you have to make?
 - d) how do results change if you survey 1,000 families? Design a simulation.
4. Ten percent of a country's population is left-handed.
 - a) What is the probability that 5 people in a group of 20 are left-handed?
 - b) What is the **expected number of left-handed people in a group of 20?**
 - c) Design a simulation to show that the expectation calculated in part b is accurate.



Medicine

5. Suppose that Bayanthisol, a new drug, is effective in 65% of clinical trials. Design a problem involving this drug that would fit a binomial distribution. Then, provide a solution to your problem.

6. Suppose that Amoxicillin, a new antibiotic, is known to completely eliminate any bacterial infection with a single dosage 95% of the time, but at a cost of \$500 a dose. A second older drug, Botosyllin, does the same job at \$100 a dose 60% of the time. A lethal strain of strep has infected a hospital ward. As hospital administrator, what would be the cheapest way to ensure you eradicated the bacterium from the 30 patients on the ward and prevented its spread?



Business Analysis (Financial Management)


7. Companies, whose shares are publicly traded, must file audited financial statements of their performance. 'Audited' means a third party has come in and verified what a firm has published. If normal practice showed that 2% of all invoices at any given firm are incorrectly recorded, in a random sample of 1,000 invoices at this firm:

- How many would an auditor expect to show irregularities?
- What is the probability of finding 40? What would you conclude if you found 40?
- If you only sampled 100 invoices and found 4, how would your answers change?

8. Lotto 649 is a game where contestants select six numbers from 1 to 49 inclusive. On the draw date, the OLG chooses six numbers at random, and one bonus number. Gamers then match their selection to the drawn numbers to determine the result. Assume there is no repetition of any kind. The payouts are shown in the table.


| Prize | Numbers Matched | Amount |
|-------|-----------------|-------------|
| 1st | 6 | \$7,200,000 |
| 2nd | 5 | \$74,000 |
| 3rd | 4 | \$2,800 |
| 4th | 3 | \$85 |
| 5th | 2 | \$10 |
| 6th | 1+Bonus | \$5 |

What must the Ontario Lotto & Gaming Corporation charge per ticket to ensure it makes a profit? Is this how much it charges?



Criminology

9. A stabbing at a Toronto night club has left police with few witnesses willing to come forward. They believe two men got into an argument inside the bar, then went outside through the back entrance into the alley where the victim was eventually stabbed. An autopsy indicated the time of death to be midnight. It also showed that the victim's blood type was A+, but he had O+ blood under his fingernails. Police managed to get the names of some of the patrons of the bar who were in the back around the time of the homicide. Otherwise, the only evidence police have to go on is a suspicious Toyota in the back alley just before the stabbing; a nearby bank ATM surveillance camera captured the make of a vehicle in the ally as it drove away at about this time. It was a 2005 Toyota Corolla.



| | |
|--------------------|---------|
| Toyota Corolla | 46,533 |
| All Light Vehicles | 408,582 |

When police interview the 5 patrons from the back of the bar, how likely is it that any of them - just by chance - would have O+ blood and drive a Toyota? Would/could investigators use this information? Could the Crown rely on this evidence in court?

| | | | |
|-----|-----|-----|----|
| O+ | 37% | O- | 6% |
| A+ | 34% | A- | 6% |
| B+ | 10% | B- | 2% |
| AB+ | 4% | AB- | 1% |



Las Vegas!

10. In the game of roulette the host spins the wheel. One of each number on the table pictured at right is on the wheel. Each time it is spun a marble lands on one and only one number. Players can bet on numbers, colour, even or odd numbers, rows, squares, or columns with varying payout ratios. The house pays 35 to 1 if you bet on a single number and your number comes up.



- With a \$100 bet on **black 17**, what is the probability of winning?
- What would you expect to win before the wheel was spun?
- How about the probability of winning an outside bet on the colour **red** which pays 1:1?
- What would you expect to win after one spin? After 10 spins? What does this tell you about casinos?



Military Strategy

A B-52 can be used for a type of aerial assault known as "carpet bombing". It is cheaper, but not as accurate as laser-guided bombs. Ordnance dropped from a B-52 on target ranges has a 0.6% chance of a direct hit. A Mark-82 (500 lb bomb) will destroy a target if it's close, but only a direct hit will take out a concrete structure. A B-52 can hold 51 Mark-82s.

A S.O.A.R. navy seal team is waiting to hit target A, specified at right, but they only have a team of 8. The concrete brigade bunker B needs to be taken out before they move in, or they will be overrun by troops during their extraction.

Can Air Force command assure the Navy that the seal team will be all clear after a single B-52 raid? How many sorties would the USAF need to send to ensure the bunker was destroyed?



Review

Binomial | Geometric | Hypergeometric

| Distribution | P(X) Probability | E(X) Expected Value | Conditions |
|----------------|--|---------------------------------|---|
| Binomial | ${}_n C_r \times p^r \times q^{n-r}$ | nP | Constant Probability Replacement |
| Geometric | $q^{n-1}p$ | $\frac{Q}{P}$ | Constant Probability Replacement |
| Hypergeometric | $\frac{{}_a C_x \cdot {}_{n-a} C_{r-x}}{{}_n C_r}$ | $r \frac{a}{n}$ (just nP) | Dependent, or Varying Probability No Replacement |