

Unit 6 Lesson 2:

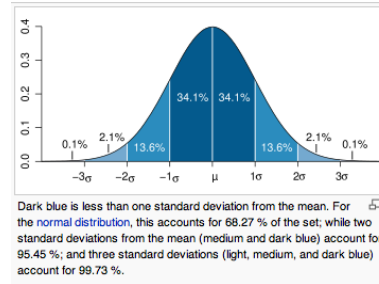
Probability Distributions and Z-Scores

Quincy has a 93% in his math class, a course that has a mean of 77%. His physics grade is 94% (where the course mean is 78%). Mathematically speaking, of which grade should he be more proud?

Is there another piece of information that would make the picture more clear?

A Quick Reminder

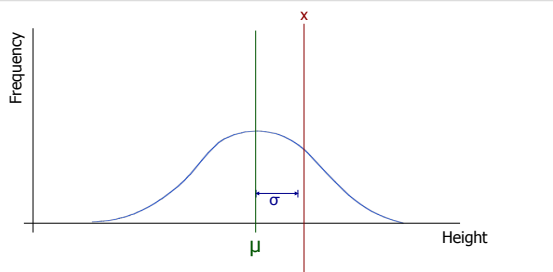
A reminder of the last lesson, courtesy of Wikipedia



The confidence intervals are as follows:

σ	68.26894921371%
2σ	95.44997361036%
3σ	99.73002039367%
4σ	99.99366575163%
5σ	99.99994266969%
6σ	99.99999980268%
7σ	99.9999999974%

The Z-Score



$$z = \frac{x - \mu}{\sigma}$$



So what IS a Z-score?

Z-Scores:

How far does data lie from the mean as a multiple of the standard deviation? (i.e. how many standard deviations away from the mean is a given data point?)

$$z = \frac{x - \mu}{\sigma}$$

data point (pointing to x)

mean (pointing to μ)

standard deviation (pointing to σ)

Essentially, the larger the absolute value of the z-score, the more extraordinary the score.

Catching Z's

- Calculate the z-scores for the following data points if the mean for the data set is 6 and the standard deviation is 2.
a) 4 b) 7 c) 1.5
- A student scores 85% on a data management test. The class average was determined to be 77% with a standard deviation of 3.2%. How many standard deviations from the mean is the students' score? Is the score above or below the mean?
- A data set has a mean of 23 and a standard deviation of 5. Determine the value of the data points corresponding to the following z-score values
a) 0.6 b) 1.5 c) -2.3
- A given data point in a set of data has a value of 7.3 and a standard deviation of 3. Determine the mean of the data set if the data point has the following z-score values
a) 2.1 b) -1.5
- Calculate the mean of a data set given the following information about the data point and the standard deviation
a) $x = 78, z = 1.56, \sigma = 8$ b) $x = 198, z = -1.3, \sigma = 23.6$

- Michael is 190cm tall. In his high school, heights are normally distributed with a mean of 165 cm and a standard deviation of 20 cm. What is the probability that Michael's best friend is shorter than he is?
- Testing has shown that new Blu-ray players have a mean lifetime of 6.2 years. Lifetimes for these Blu-ray players are normally distributed and have a standard deviation of 1.08 years. If the company offers a 5 year warranty on parts and labour, what percent of Blu-ray players will fail before the end of the warranty period?
- There have been some outstanding hitters in baseball. In 1911, Ty Cobb's batting average was 0.420. In 1941, Ted Williams batted 0.406. George Brett's 0.390 average in 1980 was one of the highest since Ted Williams. Batting averages have historically been approximately normally distributed, with means and standard deviations as shown below.

Decade	Mean	Standard Deviation
1910s	0.266	0.0371
1940s	0.267	0.0326
1970s-1980s	0.261	0.0317

Compute z-scores for each of these three outstanding hitters. Can you rank the three hitters? Explain your answer.

- To run without causing damage, diameters of engine crankshafts for a certain car model must fall between 223.92 mm and 224.08 mm. Crankshaft diameters are normally distributed, with a mean of 224 mm and a standard deviation of 0.03 mm. What percent of these crankshafts are likely to cause damage?
- The daily discharge of lead from a mine's tailings is normally distributed, with a mean level of 27 mg/L and a standard deviation of 14 mg/L. On what portion of days will the daily discharge exceed 50 mg/L?
- The average heights of teenage girls are normally distributed, with a mean of 157 cm and a standard deviation of 7 cm.
a) What is the probability that a teenage girl's height is greater than 170 cm?
b) What range of heights would occur about 90% of the time?

- Use the Standard Normal Distribution table to solve Normal Distribution problems. Include a sketch of the distribution with your solution including each of the standard deviation intervals.

A data set is normally distributed with a mean of 25 and a standard deviation of 5. Using the z-score tables determine the following probabilities for a data point chosen at random.

- a) $P(x < 25)$ b) $P(x < 20)$

- c) $P(x > 30)$

d) $P(20 < x < 30)$

e) $P(15 < x < 35)$

f) $P(17 < x < 22)$

Answer Clues

1a) -1
b) 0.5
c) -2.25

2) 2.5σ above

3a) 26
b) 30.5
c) 11.5

4a) 1
b) 11.8

5a) 65.52
b) 228.68

6) 89.4%

7) 13.3%

8) Ted, Ty, then George

9) -0.8%

10) -5%

11a) -3.2%
b) -145.5cm to 168.5cm

12a) 50%
b) -15.9%
c) -15.9%
d) -68.3%
e) -95.5%
f) -22%